TEARING INSTABILITY OF A FLAT CURRENT LAYER WITH FINITE IONIC VISCOSITY

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1. Rupture instability is a very interesting and important physical phenomenon which has been intensively studied in recent years (see, e.g., [1]). The development of this instability is affected by a series of different physical factors. Plasma flow along the current layer and the component of the magnetic field normal to the layer are significant [2]. The stability of the current layer is affected by the nonuniformity of the density and conductivity, as well as by transport of the conductivity together with the motion of the medium (convection conductivity).

One of the important factors is ionic viscosity, which can significantly affect the tearing mode. An attempt to evaluate the effect of ionic viscosity was first made in [3]. The effect of ionic viscosity on the stability of the diffusion pinch and the parallel viscosity and compressibility factors were studied in [4-8]. The asymptotic method and the solution in [3], used with some modifications in [4-8] also, involve heuristic elements already in the case of infinitesimal ionic viscosity. The asymptotic theory does not permit studying the case of "moderate" magnetic Reynolds numbers R_m, which are characteristic for the experiment. In the presence of finite ionic viscosity, additional complications appear in connection with, in particular, the increase in the order of the system of differential equations.

A correct study of the instability of the tearing mode can be performed using the methods of the theory of stability of flows of viscous liquids. Analytic methods of the theory of hydrodynamic stability were used in [9, 10] to solve a number of problems concerning the plasma stability, where it was pointed out that the processes occurring in the development of instability in a plasma and in an ordinary liquid are similar [9]. In [11] numerical methods of the theory of hydrodynamic stability were used to study the Alfven oscillations of an inhomogeneous plasma in the presence of a beam of fast ions.

We draw attention to the analogy between the instability of the tearing mode and the magnetic branch of the instability of the Hartman flow, discovered previously by one of the authors [12]. This analogy permits using the computational algorithms in [12, 13] to study the tearing mode. The numerical experiments performed give a detailed picture of the effect of ionic viscosity on the stability of the current layer.

The simplest model of a current layer is a one-dimensional static configuration of a plasma and a magentic field, studied in the MHD appproximation [3]. In this work, we study in the MHD approximation the stability of a flat current layer of a collisional plasma, bounded by well-conducting impenetrable plates at $y = \pm L_0$. The magnetic field is oriented along the x axis and its sign changes on the neutral plane $y = 0$. This magnetic field is maintained by the current flowing along the z axis.

It is convenient to use dimensionless notation. For the scale length we use the half-width of the channel, for the density scale we use the density n_0 along the channel axis $y = 0$; for the velocity scale we use the Alfven velocity $V_0 = B_0/\sqrt{4\pi n_0 m_i}$, calculated using the field scale B_0 and the density scale n_0 (m_i is the mass of the ions).

Let the magnetic field have the form [3]

$$U(y) = \operatorname{th} py. \tag{1.1}$$

The quantity p characterizes the rate of change of the magnetic field near the zero line (in the dimensional form the field $B = B_0 \tanh (y/a)$, and in addition $p = L_0/a$). Following [2, 3], we choose the density profile in the form

$$n(y) = 1/\operatorname{ch}^2 py. \tag{1.2}$$

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We shall first assume that the ions in the plasma are not magnetized. The viscosity of the plasma can then be assumed to be isotropic. This is valid if [1, 13]

$$(\omega_i\tau_i)^2 = (4.16\cdot10^{22}/AZ^4\lambda^2)(B[G])^2(T[\mathrm{eV}])^3/(n[\mathrm{cm}^{-3}])^2 \ll 1.$$

This inequality is usually satisfied in experimental setups, used to study a current layer (see, e.g., [1]). The plasma temperature in such setups is characteristicslly of the order of several tens of electron volts $B \sim 10^3$ G, $n \sim 10^{15}$–10^{16} cm^{-3} (helium and argon plasmas are often used). In stronger magnetic fields and at higher temperatures, the viscosity of the plasma is determined by tensor quantities. For T > 100 eV the field-aligned ionic viscosity plays a substantial role [4], and the transport of momentum across the magnetic field is hindered for sufficiently large values of B.

Using the results of [3], we estimate the magnetization factor, taking into account the decrease in the coefficient of viscosity in a magnetic field within the framework of the isotropic-viscosity model. In this case, in particular, the fact that the field-aligned momentum is transported along the magnetic field freely is ignored. It is therefore reasonable to use the transverse viscosity until the field-aligned viscosity becomes quite large. An exact analysis of the effect of ionic viscosity must be performed taking into account the complete viscous stress tensor of the plasma in magnetic field. This, however, is a very complicated problem and has not yet been studied.

The linear stability of the current layer at rest within the framework of the model studied will be determined by the following eigenvalue problem:

$$\begin{gathered}\frac{n(y)\,\nu(y)}{ik\,\delta\,\mathrm{R}}v^{\mathrm{IV}} + C\left[(nv')' - k^2nv\right] = -U\left(h'' - k^2h\right) + U''h,\\ h'' = -\left(ik\,\delta\,\mathrm{R}_m U\frac{U'(y)}{U'(0)} + \frac{U''(y)}{C}\right)v + \left(k^2 - \frac{U'(y)}{U'(0)}ik\,\delta\,\mathrm{R}_m C\right)h,\\ v(\pm1) = 0,\ h(\pm1) = 0.\end{gathered} \tag{1.3}$$

Here v and h are the y components of the complex amplitudes of the velocity and field perturbations; k is the wave number; $C = X + iY$ is the complex phase velocity; $\gamma = kY\delta$ is the increment (the perturbations grow, if Y > 0); $R = V_0L_0/\nu_0$ is Reynolds' number, calculated based on the magnitude of the kinematic viscosity ν_0; R_m is the magnetic Reynolds number determined from the magnitude of the electrical conductivity at y = 0; δ is the cosine of the angle between the x axis and the wave vector; and, $\nu(y)$ is the profile of the kinematic viscosity. The prime indicates differentiation with respect to y. In exactly the same manner as in [3], only the viscous term with the highest-order derivative is retained in Eq. (1.3).

According to [3], the profile of the kinematic viscosity can be taken in the form

$$\nu(y) = 1/(1 + \varphi^2U^2), \tag{1.4}$$

where φ is the magnetization parameter of the plasma. With low magnetization ($\varphi \ll 1$) the quantity ν can be assumed to be approximately constant and equal to one. In the case of large magnetization the viscosity decreases rapidly from the center of the layer to the walls.

Generalizing the well-known result of [3] to the case of a finite ionic viscosity, we find for $\nu \equiv 1$ from Eqs. (1.3) the integral relation

$$\begin{gathered}\int\limits_{-1}^{+1}\left\{-\frac{C\,|v''|^2}{i\,|C|^2\widetilde{\mathrm{R}}} + \frac{C^2}{|C|^2}\left(n\,|v'|^2 + k^2n\,|v|^2\right) + \frac{U''/U + i\widetilde{\mathrm{R}}_m CU'}{|U''/U + i\widetilde{\mathrm{R}}_m CU'|^2}\right| - h'' +\\ + \left(k^2 + U''/U\right)h\,|^2 - |h'|^2 - \left(k^2 + \frac{U''}{U}\right)|h|^2\Big\}\,dy = 0,\quad \widetilde{\mathrm{R}}_m = k\delta\mathrm{R}_m/U'(0),\\ \widetilde{\mathrm{R}} = k\delta\mathrm{R}.\end{gathered} \tag{1.5}$$

Taking the imaginary part of this relation, we arrive at the conclusion that even when the ionic viscosity is taken into account the oscillating perturbations can only decay. We shall study further the real part of (1.5). Determining $|U''/U|$ on the segment −1, +1, we conclude that for $k > \sqrt{2}p$ the monotonic perturbations (x = 0) can only decay. The numerical experiments performed show that even in the case of variable viscosity there are no growing oscillatory modes and a short-wavelength limit of the instability exists.

The increments of the monotonic perturbations are found from the following eigenvalue problem:

$$-\frac{\nu(y)}{k\delta \mathrm{R}Y}V^{\mathrm{IV}}+V''=-LV'+[k^2+(k\delta\mathrm{R}_mU^2\sigma/nY)+UK/nY]V+(U''/nY-k\delta\mathrm{R}_m\sigma U/n)h; \tag{1.6}$$

$$h''=-(k\delta\mathrm{R}_mU\sigma+K)V+(k^2+k\delta\mathrm{R}_mY\sigma)h,$$
$$\sigma(y)=U'(y)/U'(0),\ L(y)=n'(y)/n(y),\ K(y)=-U''(y)/Y; \tag{1.7}$$

$$h(\pm 1)=0,\ V(\pm 1)=0,\ V'(\pm 1)=0. \tag{1.8}$$

Here V = iv. The distribution of the conductivity is given by the function $\sigma(y)$. The quantity L characterized the nonuniformity of the density. Transport of conductivity together with the motion of the medium is taken into account with the help of the function K(y).

In the standard asymptotic theory it is assumed that $n \equiv 1$, $L \equiv 0$, $K \equiv 0$. In a narrow internal region of width ε near the zero line it is assumed that $\sigma \equiv 1$, $\nu \equiv 1$ (such a layer is said to be singular). In the outer region the finite electrical resistance, the ionic viscosity, and inertia are neglected. The perturbations are assumed to be two-dimensional ($\delta = 1$) and their wavelengths are assumed to be long ($k \ll 1$). To lower the order of the system (1.6), the following estimate is used: $V^{\mathrm{IV}} \sim V^{\mathrm{II}}/\varepsilon^2$. In addition, conditions under which the term V^{II} can be neglected in the first equation (1.6) are studied. These assumptions enable studying essentially only qualitatively the effect of ionic viscosity for large R and R_m. It is established in [3] that for sufficiently large R_m the ionic viscosity decreases the growth increments. It is, however, difficult to evaluate reliably the degree of this effect from the results of [3]. A more accurate result is obtained in [3, 15] for large R_m and $R \to \infty$. According to [3, 14], for the profile of the field (1.1)

$$\gamma = Ak^{2/5}\mathrm{R}_m^{-3/5},\quad A=\left[\frac{\Gamma(1/4)}{4\pi\Gamma(3/4)}\sqrt{p}\Delta\right]^{4/5},$$
$$\Delta = 2p^2\left[1-\frac{1}{p\,\mathrm{th}\,p}\right],\quad k^2\ll 1,\quad k\mathrm{R}_m\gg 1,\quad p>1{,}2,\quad \mathrm{R}\to\infty.$$

2. We shall establish the simplest properties of the problem (1.6)-(1.8), restricting our analysis to two-dimensional perturbations ($\delta = 1$).

For small k the quantity Y depends on the complexes kR and kR_m, and the expression for the increment has the form

$$\gamma = kY(p,\ k\mathrm{R}_m,\ k\mathrm{R}),\ k\ll 1. \tag{2.1}$$

It is evident from the formula (2.1) that if the numerical calculations are performed for fixed R and R_m in a sufficiently wide region of $k \ll 1$, then they determine the magnitude of the increments in the entire range of values of R_m and R.

We denote by γ_* the largest increment for given p, R_m, and R. From the system (1.6)-(1.8) it is easy to obtain for small R_m, kR_m the following asymptotic dependence:

$$\gamma_* = f(p,\ \mathrm{R}),\quad \mathrm{R}_m\to 0. \tag{2.2}$$

The function f(p, R) can be determined only by numerical analysis.

For small k in the system of equations (1.6)-(1.8) terms of order $\sim k^2$ can be neglected. For this reason, for fixed p, R_m, R we have the asymptotic dependence

$$Y\to \text{const as}\quad k\to 0,\ \gamma\to \text{const}\,k\ \text{ as }\ k\to 0. \tag{2.3}$$

If the nonuniformity and convection factors are neglected, then instead of (2.3) we find

$$Y\sim k\mathrm{R}_m\ \text{as}\ \ k\to 0,\ \gamma\sim k^2\mathrm{R}_m\ \text{as}\ \ k\to 0. \tag{2.4}$$

The asymptotic dependences (2.1)-(2.4) and the short-wavelength boundary of the instability restrict the region of numerical analysis to a definite range of wave numbers.

In order that the lines of force reconnect, separate elements of the plasma near the zero line of the field must move toward one another, i.e., the function v(y) must be odd (in the case of an odd field profile). The field perturbations must in this case be even functions. The boundary conditions for such perturbations have the form

$$v(0)=0,\ h'(0)=0,\ v(1)=0,\ h(1)=0. \tag{2.5}$$

We shall study qualitatively the effect of the nonuniformity and convection of the conductivity.

In accordance with the model of a singular layer, the behavior of the perturbations with large values of kR_m, kR is determined, first of all, by the distribution of the magnetic field throughout the entire thickness of the current layer and, second, by the local characteristics of the current layer near y = 0. For this reason, the nonuniformity factors, the magnetization parameter, and the convection parameter must not significantly affect the magnitude of the increments of such perturbations (near the axis of the layer, according to (1.4) and (1.7), ν, n, $\sigma \equiv 1$, L = K = 0).

The behavior of the perturbations for "moderate" values of R_m is determined only by the integral characteristics of the current layer. The effect of the nonuniformity of the conductivity has approximately the same significance for such perturbations as does a definite decrease in the value of R_m.

The drop in the density from the center to the walls of the channel, e.g., following the law (1.2), enables neglecting the inertia in the outer region. For this reason, when the density nonuniformity is taken into account, the correspondence with the asymptotic theory for large R and R_m [3, 15] should improve. The nonuniformity factor will in this case have a definite destabilizing effect. For moderate values of R and R_m, this factor can have a different effect.

Elements of the medium moving toward the zero line decrease the conductivity in the vicinity of this line, thereby easing the conditions of reconnection. For this reason, the conductivity convection should probably have a destabilizing effect.

Taking into account the magnetization parameter has approximately the same significance as a definite increase in R. This is evident directly from Eqs. (1.6) and the relation (1.4). An increase in the parameter φ must weaken the effect of the ionic viscosity. It should be expected that the magnetization should evidently have an effect only when $\varphi^2 U^2(\varepsilon) >> 1$. For the magnetic-field profile (1.1) this inequality, taking into account the smallness of ε, is represented in the form

$$\varphi^2 p^2 \varepsilon^2 \gg 1.$$

3. The equations (1.3) contain, even for "moderate" values of R, R_m, and φ, small parameters in front of the higher order derivatives of the functions v and h. Under these conditions, a special region is separated in the current layer near the neutral plane, which transforms into a narrow singular layer when R and R_m increase. In addition, characteristic "viscous" regions exist near the walls. This problem is somewhat reminiscent of classical problems in the theory of hydrodynamic stability of flows of a viscous liquid.

Using the above-noted similarity between the tearing instability and the magnetic branch of the instability of Hartman's flow, we propose using the computational methods presented in [12, 13]. As is well known, the choice of the iteration scheme is not unique. In this work, we use the optimal variant

$$\begin{pmatrix} v'' \\ v''' \\ h' \end{pmatrix} = \begin{pmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{pmatrix} \begin{pmatrix} v \\ v' \\ h \end{pmatrix} \tag{3.1}$$

proposed in [13].

The starting linear system is integrated first at a small distance from the channel wall δ_0. The value of δ_0 was usually chosen to be equal to 10^{-3}. Three solutions of the fundamental system, satisfying the boundary conditions, are found. With the help of these solutions, at $y = 1 - \delta_0$ the initial data for the nonlinear system of equations, which the quantities A_{ij}(i, j = 1, 2, 3) from the relation (3.1) satisfy, are calculated. At y = 0, using the boundary conditions, we obtain from the relations (3.1) the characteristic equation

$$F(y) = A_{12}A_{33} - A_{13}A_{32} = 0 \tag{3.2}$$

for the even perturbations. The roots of the characteristic equation (3.2) were first found with the help of a grid of values of the function F. After finding the "reference" values of Y, the iterative method of secants, which is usually used in hydrodynamic problems [16], was used. The step with "motion along the continuous line" [16] was determined experimentally.

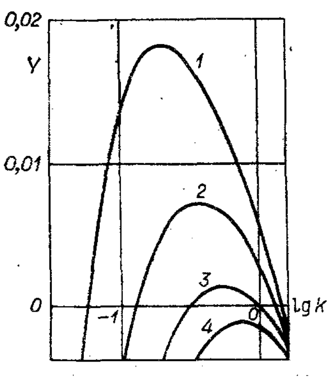

Fig. 1

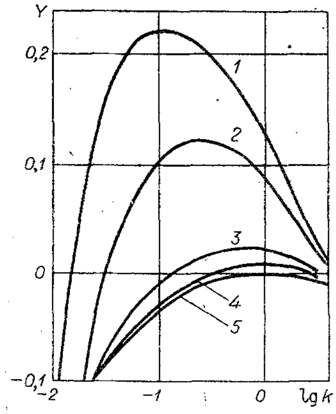

Fig. 2

When the quantity k was changed, the step was most often chosen to be equal to 5%. An analysis of the behavior of the function F(Y) indicates the existence of only one root for the physical parameters studied here.

The numerical calculations were performed on the ES 1022 computer (double precision was used). The eigenvalues were found with fixed accuracy (three significant figures). The numerical experiments were performed in a wide range of physical parameters: $p = 1$-10, $k = 10^{-4}$-10, $R_m = 1$-10^5, $R = 1$-10^6, $\varphi = 1$-8.

4. We shall first study two-dimensional perturbations neglecting the nonuniformity, convection, and magnetization factors. In this case $\sigma \equiv 1$, $\nu \equiv 1$, $L \equiv 0$, $K \equiv 0$.

The numerical experiments show that when the ionic viscosity increases, the tearing instability can be significantly or even completely suppressed.

Figure 1 demonstrates the stabilization of the tearing instability as the ionic viscosity increases for $p = 2$, $R_m = 10^3$ ($R = 10^3$ (1); 200 (2); 40 (3); 10 (4)). At $R = 10$ the tearing instability is completely suppressed.

The stabilization picture presented here is generally typical for small p. Thus for small p the ionic viscosity may affect the tearing mode. The stabilization observed here is not described, even qualitatively, by the elementary theory [3]. We note that the similarity criteria which we used are not exotic for experiments. For example, at a temperature of $T \sim 30$ eV, $n \sim 10^{16}$ cm^{-3}, $B \sim 10^3$ G and $L_0 \sim 10$ cm, we have $R \sim 30$.

The ionic viscosity has an especially strong effect on the perturbations with small growth increment. As can be seen from Fig. 1, even with a low ionic viscosity the long-wavelength perturbations with a small growth increment are suppressed. For this reason, when the ionic viscosity is taken into account, a long-wavelength instability boundary appears together with the short-wavelength instability boundary. It is interesting to note that when the ionic viscosity increases, not only do the growth increments decrease, but the region of wave numbers corresponding to growing perturbations becomes narrower also.

Figure 2 shows the effect of the ionic viscosity for $p = 6$, $R_m = 10^3$ ($R = 10^3$ (1); 200 (2); 10 (3); 2 (4); 0.4 (5). The curves shown illustrate the changes occurring when p is increased. As p increases (for fixed R and R_m), the increments grow. The ionic viscosity, as before, has a significant stabilizing effect, but total stabilization appears for very small values of R. For $p = 6$ all perturbations decay, if $R < 0.4$. But, for such small R, the starting equations (1.3) are, generally speaking, inapplicable.

In the linear theory it is of greatest interest to determine the maximum increment

$$\gamma_* = \max_k \gamma(k, R_m, R, p).$$

The typical dependences $\gamma_*(R)$ for $p = 2$ are shown in Fig. 3 ($R_m = 10$ (1); 10^2 (2); 10^3 (3)). For very large R the ionic viscosity does not significantly affect the stability of the current layer. The limiting values of γ_* in the limit $R \to \infty$ are already actually achieved when $R = 10^5$. In the cases studied in Fig. 3, for very large R, perturbations with $R_m = 10^2$ have the highest increments. However, when the ionic viscosity is increased, these increments drop rapidly and for sufficiently small R the perturbations with $R_m = 10^3$ have the largest increments. As can be seen from Fig. 3, these perturbations are completely stabilized only when the ionic viscosity has a significant magnitude.

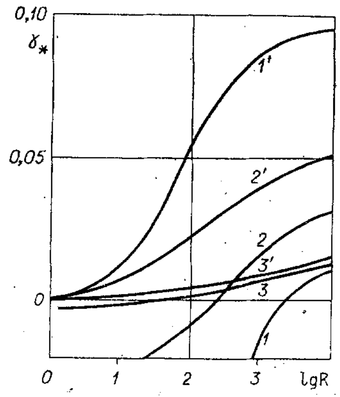

Fig. 3

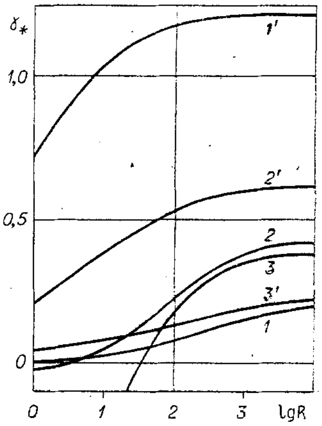

Fig. 4

The characteristic dependences $\gamma_*(R)$ for $p = 6$ are presented in Fig. 4 ($R = 10$ (1); 10^2 (2); 10^3 (3)). These curves demonstrate the changes occurring when p is increased. Although the increments grow significantly when p increases, the ionic viscosity has an effective stabilizing action even in these cases.

We shall evaluate the role of three-dimensional perturbations, i.e., inclined perturbations with $\delta < 1$. Let the values $R_m = R_{m0}$ and $R = R_0$ be given. Then, it is evident directly from Eqs. (1.6) that the increments of the three-dimensional perturbations can be found if the solution of the two-dimensional problem is known for all $R_m < R_{m0}$ and for fixed ratio $P_m = R_m/R = R_{m0}/R_0$ (the quantity P_m can be called the magnetic Prandtl number). Thus the starting three-dimensional problem reduces to an equivalent two-dimensonal problem. This property is analogous to Squire's transformation in hydrodynamics.

Figure 5 shows the dependences $\gamma_*(R_m)$, constructed with $p = 6$ and $P_m = 0.25$, 1, and 4 (curves 1-3). The broken line shows the limiting dependence $\gamma_*(R_m)$, obtained in the limit $R \to \infty$. It is evident from Fig. 5 that the effectiveness of the stabilization increases as the quantity P_m decreases.

Let $R_m = R_{m0}$ be given. Then in order to determine the maximum increment it is necessary to find $\delta\gamma_*(R_m)$ directly from Fig. 5 for the values $R_m < R_{m0}$. From here the slope angle of the wave vector of the perturbation with highest increment is also found at the same time. This analysis shows that straight perturbations ($\delta = 1$) have the highest increments.

In real finite systems it is necessary to take into account the fact that the quantity $k\delta = k_x > 2\pi/L_x$ (L_x is the dimension of the system along the x axis).

5. In a nonuniform magnetized plasma, the increments undergo substantial changes, especially in the long-wavelength region. For large R_m, however, in the range of wave numbers corresponding to the largest increments, the quantities γ_* vary quite insignificantly. This means that for large R_m a narrow singular layer, within which the nonuniformity and magnetization factors can be neglected, is separated in the plasma. For comparatively small R_m, however, a distinct singular layer does not exist, so that the increments depend substantially on the nonuniformity, convection, and magnetization factors.

Figure 6 shows the typical dependences $Y(k)$, demonstrating the effect of the above-indicated factors with $p = 6$, $R = 10^3$, $R_m = 10^3$. Curve 1 shows the dependence $Y(k)$, obtained taking into account all factors, except the magnetization factor. The curve 2 was calculated taking into account the nonuniformity of the conductivity and the convection conductivity only. As also in the case when ionic viscosity is absent, the indicated factors significantly increase the increments in the long-wavelength region. The curve 3 represents the dependence $Y(k)$, obtained taking into account only the magnetization factor with $\varphi = 5$. Curve 5 was constructed taking into account only the nonuniformity of the density, and curve 6 was constructed taking into account only the nonuniformity of the conductivity. For comparison, Fig. 6 shows the curve $Y(k)$ for the uniform problem (curve 4).

The nonuniformity of the density, in this case, has the weakest effect on the increments. We note that when the convection conductivity is taken into account (together with the nonuniformity of the conductivity), complete stabilization does not occur in the long-wavelength region. Thus in the plasma with a large gradient of the conductivity the effect of ionic viscosity is weakened.

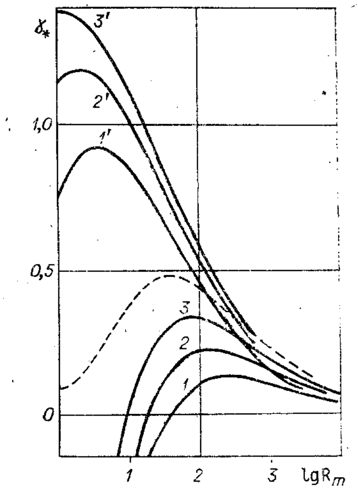

Fig. 5

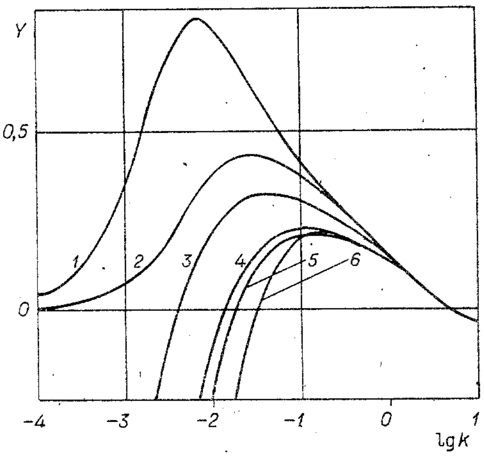

Fig. 6

The curves $\gamma_*(R)$ taking into account all nonuniformity, convection, and magnetization factors ($\varphi = 5$) with p = 2 are shown in Fig. 3 ($R_m = 10$ (1'); 10^2 (2'); 10^3 (3')). In this case complete stabilization does not occur, but for small R the increments are small. As is evident from Fig. 3, the smaller the value of R_m, the more rapidly the limiting values of γ_* are reached with large R.

The cases of large p are shown in Fig. 4. Graphs of the functions $\gamma_*(R)$ with p = 6 are shown here taking into account all nonuniformity and convection factors ($R_m = 10$ (1'); 10^2 (2'); 10^3 (3')). As in the preceding example, the magnetization parameter $\varphi = 5$.

Figure 5 shows the curves $\gamma_*(R_m)$ for fixed P_m ($P_m = 0.25$ (1'); 1 (2'); 4 (3')) taking into account all convection and magnetization nonuniformity factors ($\varphi = 5$). The aggregate effect of these factors leads to a significant increase in the increments with comparatively small R_m. However, for large R_m, the effect of these factors is insignificant.

Let $R_m = R_{m_0}$ be given. Moving along the curves studied in the direction of decreasing R_m from R_{m_0}, we find the increments of the three-dimensional perturbations for different slope angles.

In the experiments, the plasma current layer with the parameters studied here exists for a time much longer than the magnitude of the inverse increment, determined in the limit $R \to \infty$ in the static model. In this work it was established that the ionic viscosity can significantly decrease the increments of the rupture instability, and in certain situations completely stabilize it. Thus, in order to explain the laboratory experiments performed with a quite dense and "moderately" heated plasma in a comparatively weak magnetic field, the ionic viscosity factor must be taken into account (together with other stabilizing factors), especially in those cases when the Reynolds number is comparatively small. For large R, however, the ionic viscosity has an insignificant effect.

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STRUCTURE OF AN AXISYMMETRICAL NONSTATIONARY WAVE OF ABSORPTION OF LASER RADIATION IN A TRANSPARENT DIELECTRIC

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Thermal laser breakdown of an initially transparent dielectric and the subsequent formation of a plasma wave of absorption of radiation in it, in contrast to the well-studied analogous phenomena in gases [1], have been studied comparatively little. The reason for this is the large number of physical phenomena occurring, as well as the lack of the exact values of the quantities characterizing the state of the dielectric in the pre and post-breakdown states. A comparison of the experimental results [2, 3], theoretical estimates [4, 5], and one-dimensional numerical calculations [6, 7] indicates that the appearance and propagation of the thermal absorption wave is satisfactorily described by the mechanism of nonlinear heat conduction with the appropriate coefficients of thermal conductivity and absorption of laser radiation. At this stage the main parameters of the absorption wave are determined: the propagation velocity, the average and maximum temperatures, and the thickness of the front. The motion of the plasma formed, the possibility of the occurrence of dissociation processes, the presence of defects which absorb radiation, the effect of the dielectric outside the thermal wave, and some other effects are not studied.

This paper is concerned with the study of the effect of two-dimensionality on the thermal wave in a dielectric within the framework of the physical models developed previously for the one-dimensional and nonstationary cases [4, 6, 7].

The following system of equations was studied numerically:

$$c\frac{\partial T}{\partial t}=\frac{\partial}{\partial x}\varkappa(T)\frac{\partial T}{\partial x}+\frac{1}{r}\frac{\partial}{\partial r}r\varkappa(T)\frac{\partial T}{\partial r}+k(T)q,\quad \frac{\partial q}{\partial x}=k(T)q, \tag{1}$$

where T is the temperature; $\varkappa(T)$ is the coefficient of thermal diffusivity; q is the power density of the radiation; k(T) is the absorption coefficient; and c is the heat capacity of the medium, assumed to be independent of the temperature. The coefficients $\varkappa(T)$ and k(T) were taken from [6]:

$$\begin{gathered}k(T)=k_0+k_1\exp(-E/2T),\ \varkappa(T)=\varkappa_0+\varkappa_1 T\exp(-E/2T),\\ k_0=0.25\ \text{cm}^{-1},\ k_1=5\cdot 10^4\ \text{cm}^{-1},\ \varkappa_0=1.5\cdot 10^{-}\ \text{W/(cm}\cdot\text{deg)},\\ \varkappa_1=2.9\cdot 10^{-4}\ \text{W/(cm}\cdot\text{deg)},\quad E=44000\,^\circ\text{K},\ c=3.1\ \text{J/(cm}^3\cdot\text{deg)}.\end{gathered} \tag{2}$$

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